International Conference Mathematical and Computational Modelling in Science and Technology
ICMCMST'2015, August 02-07, 2015. Izmir University-Turkey

Interaction of sine-Gordon solitons in the model with attracting impurities and damping

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Abstract: The dynamics of nonlinear waves of the sine-Gordon equation with a spatially modulated periodic potential and damping are studied using analytical and numerical methods. The structure and properties of multisolitons excited on attracting impurities are determined. For small-amplitude oscillations, an analytical spectrum of the oscillations is obtained, which is in qualitatively agreement with the numerical results.

Keywords: sine-Gordon equation; kink; multisolitons; structure and properties of solitons; numerical-analytical study; impurity.

MSC 2010: 35C08; 35Q51; 65M06.

1 Introduction

In recent decades the solitons theory for nonlinear evolutionary equations has been widely used in many fields of physics. For example, the sine-Gordon equation solitons simulate different localized dynamic excitations of the physical systems [1]. However, for an adequate description of real physical processes one requires a modification of the sine-Gordon equation by addition of summands describing the presence of damping and external force in the system, spatial modulation of the periodical potential (or impurity) Advanced analytical methods for studying this problem for the modified sine-Gordon equation (MSGE) using perturbation theory for solitons tend not to give an exhaustive result. The research of the large disturbances influence on the MSGE solution, in general case, can be conducted only with the help of numerical methods [2, 3].

2 Main results

We investigated the interaction of sine-Gordon solitons in the model with attracting impurities and damping. For the case of two impurities in the system we presented, with the help of numerical simulation, the possibility of the multisolitons states generation (such as tritons and quadrons) localized on the impurities. The following ways of the kink dynamics were found: the kink is pinned in the impurity area and oscillates in between for a certain time; it is reflected in the reverse direction or passes the impurity area. In the latter two cases oscillated localized high amplitude non-linear breather waves greatly influencing the kink energy are excited. Further interaction with the localized waves in question forms the basis for the resonance mechanism, namely "reflection from the attracting impurity". Pinning of the kink and exciting high amplitude localized non-linear waves on the

impurity may be used for multisoliton excitement in the sine-Gordon equation. A triton consisting of the weakly bound kink and breather is observed at long distances between the impurities. Starting with a certain critical distance pulsation and transmission mode frequencies are synchronized with the breather oscillation frequency and a triton solution of a wobble type is observed. At very short impurity distances excitement of the strongly bound kink and soliton is possible. The dependence of the structure and excited multisoliton frequencies from the impurity distances is determined.

There is found a definite critical distance value between the impurities that provides for two possible variants of dynamic behavior of the kink. In the first one the kink behavior is similar to the behavior in a single impurity. In the second case variants of the final kink behavior change depending on the initial kink velocity. It is explained by the breather oscillation phases excited in the second impurity area. A definite critical value of impurity distances causing two quite different ways of the dynamic kink behaviour is demonstrated. The interaction of the kink with breather can lead to a resonant "quasitunneling" of the kink through the double impurity region (i.e. passing through a potential barrier with a sub-barrier initial velocity).

Using the collective coordinates method we showed, that the original problem can be reduced to a system of ordinary differential equations for two bound harmonic oscillators with an elastic type of bind, which qualitatively describes the localized waves fluctuations. Within the limit of small amplitude fluctuations of localized waves, the spectrum of possible modes, which with good accuracy corresponds with the numerical results, was calculated.

We show analytically and numerically that the damping and external force counteract the generation of kink resonant reflection from the attracting impurity. However, its cause – resonant energy exchange between solitons – still exists. For detecting reflection resonance effects and quasitunneling in real physical experiments, we proposed a method of measuring the amplitude of translational vibrations localized in kink impurity region.

References

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