

One and n-dimensional fractional operators: Some open problems

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Abstract: As it is well known that in the literature we can find several 1-dimensional operators which have been used mainly to produce fractional model to simulate the dynamics of many process through complex media in may areas of the Applied Science and the Engineering. However many of the real problems can be find in 2 or 3-dimensional spacial media. Therefore, we will consider during our lecture some explicit n-dimensional fractional operators which extend the corresponding 1-dimensional fractional operators. Also we will try to give some justification of the connections of the fractional non-local operators and the characterization of some complex media. Finally we will try to show some of the open problems in this topic.

Keywords: Fractional Models; Anomalous Dynamics; Fractional Differential Equations; Fractional Vector Calculus.

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