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On a fractional reaction diffusion system with a balance law

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Abstract: The reaction diffusion system with anomalous diffusion and a balance law

$$u_t + (-\Delta)^{\alpha/2} u = -f(u,v),\ x \in \mathbb{R}^N,\ t > 0,$$

$$v_t + (-\Delta)^{\beta/2} v = f(u,v),\ x \in \mathbb{R}^N,\ t > 0,$$

$N \geq 1, 0 < \alpha, \beta \leq 2$, is considered. The existence of global solutions is proved in two situations:
(i) a polynomial growth condition is imposed on the reaction term $f(u,v)$ when $0 < \alpha \leq \beta \leq 2$;
(ii) no growth condition is imposed on the reaction term $f(u,v)$ when $0 < \beta \leq \alpha \leq 2$.

The nonlocal operator $(-\Delta)^{\alpha/2}, 0 < \alpha < 2$ accounts for anomalous diffusion and can be defined via the Fourier transform pair $\mathscr{F}$ and $\mathscr{F}^{-\infty}$ as

$$(-\Delta)^{\alpha/2} u(x) = \mathscr{F}^{-\infty}\left(|\xi|\mathscr{F}(u)(\xi)\right)(x),\ u \in \mathscr{S}(\mathbb{R}^N).$$

A typical type of system under our consideration is given by the irreversible molecular combination

$$mU + nV \ \rightharpoonup\ (n+1)V$$

where U and V are two chemical species. If u and v represent the concentrations of the species U and V, respectively, then according to the law of mass action of Gulberg and Waage, the reaction diffusion system describing the chemical reaction is the previous one with $f(u,v) = u^m v^n$.

Keywords: Reaction diffusion system; Fractional Differential Equations; balance law.

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