

## On a fractional reaction diffusion system with a balance law

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**Abstract:** The reaction diffusion system with anomalous diffusion and a balance law

$$u_t + (-\Delta)^{\alpha/2} u = -f(u, v), \quad x \in \mathbb{R}^N, \quad t > 0,$$

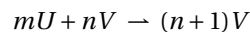
$$v_t + (-\Delta)^{\beta/2} v = f(u, v), \quad x \in \mathbb{R}^N, \quad t > 0,$$

$N \geq 1, 0 < \alpha, \beta \leq 2$ , is considered. The existence of global solutions is proved in two situations:  
(i) a polynomial growth condition is imposed on the reaction term  $f(u, v)$  when  $0 < \alpha \leq \beta \leq 2$ ;  
(ii) no growth condition is imposed on the reaction term  $f(u, v)$  when  $0 < \beta \leq \alpha \leq 2$ .

The nonlocal operator  $(-\Delta)^{\alpha/2}, 0 < \alpha < 2$  accounts for anomalous diffusion and can be defined via the Fourier transform pair  $\mathcal{F}$  and  $\mathcal{F}^{-\infty}$  as

$$(-\Delta)^{\alpha/2} u(x) = \mathcal{F}^{-\infty} (|\xi|^\alpha \mathcal{F}(u)(\xi))(x), \quad u \in \mathcal{S}(\mathbb{R}^N).$$

A typical type of system under our consideration is given by the irreversible molecular combination



where  $U$  and  $V$  are two chemical species. If  $u$  and  $v$  represent the concentrations of the species  $U$  and  $V$ , respectively, then according to the law of mass action of Gulberg and Waage, the reaction diffusion system describing the chemical reaction is the previous one with  $f(u, v) = u^m v^n$ .

**Keywords:** Reaction diffusion system; Fractional Differential Equations; balance law.

**MSC 2010:** 26A33.

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